$$u_i = B(\Phi) + \bar{\omega}_i\left[\delta_i D_1^{(i)} B(\Phi) + \varkappa_i T_1^{(i)} B(\Phi)\right] \tag{8}$$

We have retained here the notation used in formula (5). We obtain the quantities $\varkappa_i$ and δ_i from condition (7) as follows:

$$-\sigma_i\left[D_1^{(i)}B(\Phi) + \delta_i D_1^{(i)}B(\Phi) + \varkappa_i T_1^{(i)}B(\Phi)\right] + \gamma_i T_1^{(i)}B(\Phi) =$$
$$-\sigma_i\left[D_1^{(i)}B(\Phi) + \delta_j D_1^{(i)}B(\Phi) + \varkappa_j T_1^{(i)}B(\Phi)\right] + \gamma_j T_1^{(i)}B(\Phi)$$
$$(D_1^{(i)}B = -D_1^{(j)}B,\ T_1^{(i)}B = -T_1^{(j)}B)$$

and hence

$$\frac{\sigma_i}{\sigma_j} = \frac{1+\delta_j}{1+\delta_i} \quad \left(0 \leqslant M \leqslant \min\left\{\frac{2}{\sigma_l}\right\},\ \ l = 1, 2, \ldots, L\right)$$
$$-\sigma_i\varkappa_i + \gamma_i = -\sigma_j\varkappa_j + \gamma_j, \quad \varkappa_i = \frac{-N+\gamma_i}{\sigma_i}, \quad \varkappa_j = \frac{-N+\gamma_j}{\sigma_j}$$
$$(-1 \leqslant \varkappa_l \leqslant 1,\ \ \min(\gamma_l - \sigma_l) \leqslant N \leqslant \min(\gamma_l + \sigma_l),\ \ l = 1, 2, \ldots, L)$$

Formula (8) enables us to apply the proposed method to the solution of the class of problems described in /4/.

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STATE OF STRESS OF A PLANE WITH A PERIODIC SYSTEM OF PARALLEL PAIRS OF LONGITUDINAL SHEAR CRACKS*

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A closed solution of the problem of a periodic system of parallel pairs of collinear longitudinal shear cracks is obtained by the method of triple integral equations. The case of one crack of finite length in a band of periods was examined in /1-5/ for different states of stress, and of two semi-infinite cracks in /6,7/. The problem of two collinear cracks in an infinite medium was investigated in /8-11/.

Let an unlimited elastic plane xOy be weakened by a periodic system of slits $a \leqslant |x| \leqslant b$, $y = (2n+1)d$, $n = 0, \pm 1, \pm 2 \ldots$ The relationships /12/

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0, \quad \sigma_{xz} = \mu\frac{\partial w}{\partial x}, \quad \sigma_{yz} = \mu\frac{\partial w}{\partial y} \tag{1}$$

should be satisfied outside the slits., where μ is the shear modulus, w is the displacement along the z axis, and σ_{xz} and σ_{yz} are stress tensor components. We assume that the displacement and stress are periodic functions of the y with period $2d$. Then the problem reduces to constructing the solution of (1) in the strip $-d < y < d$ that satisfies the boundary conditions

$$w(x, \pm d) = 0,\ |x| \leqslant a,\ |x| \geqslant b \tag{2}$$
$$\sigma_{yz}(x, \pm d) = -T(x), \quad a < |x| < b \tag{3}$$

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We assume that the load $T(x)$ is a symmetric function of the x coordinate. We can then limit the consideration to the domain $x \geqslant 0$ and represent the solution of (1) in the form

$$w = \int_0^\infty \frac{1}{q} f(q) \operatorname{sh} qy \cos qx \, dq \tag{4}$$

$$\sigma_{yz} = \mu \int_0^\infty f(q) \operatorname{ch} qy \cos qx \, dq$$

$$\sigma_{xz} = \mu \int_0^\infty f(q) \operatorname{sh} qy \sin qx \, dq$$

To determine the function $f(q)$ from the boundary conditions (2) and (3) we obtain the triple integral equations

$$\int_0^\infty f(q) \operatorname{sh} qd \sin qx \, dq = 0, \quad 0 < x < a, \quad x > b \tag{5}$$

$$\int_0^\infty f(q) \operatorname{ch} qd \cos qx \, dq = -\frac{T(x)}{\mu}, \quad a < x < b$$

where the relationship (2) is differentiated with respect to x. We note that the solution of analogous dual integral equations is known /13/.

To construct the solution of (5), we expand the function $f(q)$ in a Kantorovich-Lebedev integral /14/

$$f(q) = \int_0^\infty A(s) K_{i2qd/\pi}(s) \, ds \tag{6}$$

Substituting the representation (6) into (5) and using the value of the integral /14/

$$\int_0^\infty K_{it}(s) \cos \alpha t \, dt = \frac{\pi}{2} e^{-s \operatorname{ch} \alpha}, \quad |\operatorname{Im} \alpha| < \frac{\pi}{2} \tag{7}$$

we obtain triple integral equations for the function $A(s)$

$$\int_0^\infty A(s) \sin sp \, ds = 0, \quad 0 < p < a_1, \quad p > b_1 \tag{8}$$

$$\int_0^\infty A(s) \cos sp \, ds = -\left(\frac{2}{\pi}\right)^2 \frac{d}{\mu} F(p), \quad a_1 < p < b_1 \tag{9}$$

$$\left(p = \operatorname{sh} \frac{\pi x}{2d}, \quad a_1 = \operatorname{sh} \frac{\pi a}{2d}, \quad b_1 = \operatorname{sh} \frac{\pi b}{2d}, \quad F(p) = T(x)\right)$$

We seek the function $A(s)$ in the form

$$A(s) = \int_{a_1}^{b_1} \varphi(t) \sin st \, dt \tag{10}$$

Taking account of the orthogonality of the functions $\sin s\alpha$ and $\sin s\beta$ for $\alpha \neq \beta$ and the formula /15/

$$\int_0^\infty \sin qx \, dq = \frac{1}{x}$$

it can be shown that (8) is satisfied identically, while we obtain an equation to determine the functions $\varphi(t)$ from (9)

$$\int_{a_1}^{b_1} \frac{t\varphi(t)}{t^2 - p^2} \, dt = -\left(\frac{2}{\pi}\right)^2 \frac{d}{\mu} F(p)$$

whose solution has the form /16/

$$\varphi(t) = \left(\frac{2}{\pi}\right)^4 \frac{d}{\mu} \frac{1}{\sqrt{(t^2 - a_1^2)(b_1^2 - t^2)}} \left[\frac{\pi C}{2d} + \int_{a_1}^{b_1} \frac{\sqrt{(p^2 - a_1^2)(b_1^2 - p^2)}}{p^2 - t^2} pF(p) \, dp\right] \tag{11}$$

where C is a constant whose value will be determined below while the integral in (11) is understood in the principal value sense. Equations (8) and (9) were examined in /17-19/ by other methods.

The stress distribution is expressed directly in terms of the function $\varphi(t)$. Substituting relationship (6) into (4) and using (7), we obtain

$$\sigma_{yz} + i\sigma_{xz} = \left(\frac{\pi}{2}\right)^2 \frac{\mu}{d} \int_0^\infty A(s)\, e^{-s\xi} ds, \quad \xi = \operatorname{ch}\frac{\pi(x+iy)}{2d} \tag{12}$$

It follows from (10) and (12) that

$$\sigma_{yz} + i\sigma_{xz} = \left(\frac{\pi}{2}\right)^2 \frac{\mu}{d} \int_{a_1}^{b_1} \frac{t\varphi(t)\, dt}{t^2+\xi^2} \tag{13}$$

We obtain a representation for the displacement in an analogous manner

$$w = \frac{\pi}{4} \operatorname{Im} \int_{a_1}^{b_1} \frac{\varphi(t)}{\sqrt{t^2+1}} \ln\left(\frac{\sqrt{t^2+1}\,\xi + t\zeta}{\sqrt{t^2+1}\,\xi - t\zeta}\right) dt, \quad \zeta = \operatorname{sh}\frac{\pi(x+iy)}{2d}$$

Substituting the function $\varphi(t)$ from (11) into (13), we obtain the final form of the distribution function.

Expressions for the stress intensity factors K_a and K_b at the points $x=a$ and $x=b$ follow from (11) and (13) and the asymptotic expression for the stress on the continuation of the crack $\sigma_{yz} \simeq K/\sqrt{2\Delta}$, where Δ is the distance from the crack apex. These expressions contain the constant C whose value should be determined from the condition that the displacement vector is single-valued during traversal along the crack contour /12/. It can be shown that this condition is expressed in terms of the function $\varphi(t)$ as follows

$$\int_{a_1}^{b_1} \frac{\varphi(t)\, dt}{\sqrt{t^2+1}} = 0$$

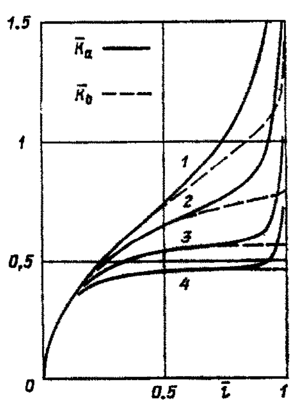
In the case of a homogeneous load $T(x) = \tau_0$, we hence obtain

$$C = -\frac{\tau_0 d}{2}\left[a_1^2 + b_1^2 - 2a_1^2 \frac{\Pi(n,k)}{K(k)}\right]$$
$$n = 1 - \frac{a_1^2}{b_1^2}, \quad k^2 = \frac{n}{1+a_1^2}$$

where $K(k)$ and $\Pi(n,k)$ are the complete elliptic integrals of the first and third kinds, respectively. In the case of a homogeneous load, we have for the stress intensity factors

$$\frac{K_a}{K_0} = \bar{K}_a = \sqrt{\frac{2d}{\pi l_0}\operatorname{th}\left(\frac{\pi a}{2d}\right)} \frac{a_1}{\sqrt{b_1^2 - a_1^2}} \left[\frac{\Pi(n,k)}{K(k)} - 1\right]$$
$$\frac{K_b}{K_0} = \bar{K}_b = \sqrt{\frac{2d}{\pi l_0}\operatorname{th}\left(\frac{\pi b}{2d}\right)} \frac{b_1}{\sqrt{b_1^2 - a_1^2}} \left[1 - \frac{a_1^2 \Pi(n,k)}{b_1^2 K(k)}\right]$$
$$(2l_0 = a + b, \quad K_0 = \tau_0 \sqrt{l_0})$$

The dependences of $\bar{K}_a$ (solid lines) and $\bar{K}_b$ (dashes) on the parameter $\bar{l} = l/l_0$ are shown in the figure, where $2l = b - a$ is the length of an individual crack. The values $d/l_0 = \infty$ (two collinear cracks), $1, 1/2$ and $1/3$ correspond to the curves *1,2,3,4*. As the crack length increases the stress intensity factors increase monotonically. When the collinear cracks merge, the factor K_a becomes infinite, while K_b remains finite. For small d/l_0 the curves $\bar{K}_a(\bar{l})$ and $K_b(\bar{l})$ have a shallow section along which the values of K_a and K_b are practically constant and equal to $\tau_0\sqrt{2d/\pi}$, the stress intensity factor for a system of parallel semi-infinite cracks.



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